The Astrophysical Journal, 545:100–106, 2000 December 10

PAIR PLASMA DOMINANCE IN THE PARSEC-SCALE RELATIVISTIC JET OF 3C 345

Kouichi Hirotani and Satoru Iguchi
National Astronomical Observatory, Osawa, Mitaka 181-8588, Japan
Moritaka Kimura[1]
Institute of Astronomy, University of Tokyo, Osawa, Mitaka 181-8588, Japan
and
Kiyoaki Wajima
Institute of Space and Astronautical Science, Yoshino-dai, Sagamihara, Kanagawa 229-8510, Japan
Received 1998 November 10; accepted 2000 July 25

ABSTRACT

We investigate whether a parsec-scale jet of 3C 345 is dominated by a normal plasma or an electron-positron plasma. We present a general condition that a jet component becomes optically thick for synchrotron self-absorption by extending the method originally developed by Reynolds and coworkers. The general condition gives a lower limit of the electron number density, with the aid of the surface brightness condition, which enables us to compute the magnetic field density. Comparing the lower limit with another independent constraint for the electron density that is deduced from the kinetic luminosity, we can distinguish the matter content. We apply the procedure to the five components of 3C 345 (C2, C3, C4, C5, and C7) of which angular diameters and radio fluxes at the peak frequencies were obtainable from literature. Evaluating the representative values of Doppler beaming factors by their equipartition values, we find that all the five components are likely dominated by an electron-positron plasma. The conclusion does not depend on the lower cutoff energy of the power-law distribution of radiating particles.

Subject headings: galaxies: active — quasars: individual (3C 345) — radio continuum: galaxies

1. INTRODUCTION

The study of extragalactic jets on parsec scales is astrophysically interesting in the context of the activities of the central engines of active galactic nuclei (AGNs). In particular, a determination of their matter content would be an important step in the study of jet formation, propagation, and emission. The two main candidates are a "normal plasma" consisting of protons and relativistic electrons (for numerical simulations of shock fronts in a VLBI jet, see Gómez, Alberdi, & Marcaide 1993, 1994a, 1994b) and a "pair plasma" consisting only of relativistic electrons and positrons (for theoretical studies of two-fluid concept, see Sol, Pelletier, & Asséo 1989; Pelletier & Sol 1992; Despringre & Fraix-Burnet 1997). Distinguishing between these possibilities is crucial for understanding the physical processes occurring close to the central "engine" (presumably a supermassive black hole) in the nucleus.

VLBI is uniquely suited to the study of the matter content of pc-scale jets because other observational techniques cannot image at milliarcsecond resolution and must resort to indirect means of studying the active nucleus. Recently, Reynolds et al. (1996) analyzed historical VLBI data of the M87 jet at 5 GHz (Pauliny-Toth et al. 1981) and concluded that the core is probably dominated by an $e^{\pm}$ plasma. In the analysis they utilized the standard theory of synchrotron self-absorption to constrain the magnetic field, B (G), and the proper number density of electrons, N_e^* (1 cm^3), of the jet and derived the following condition for the core to be optically thick for self-absorption: $N_e^* B^2 > 2\delta_{\rm max}^{-2}$, where $\delta_{\rm max}$ refers to the upper limit of the Doppler factor of the bulk motion. This condition is, however, applicable only for the VLBI observations of M87 core at the 1972 September and 1973 March epochs. Therefore, in order to apply the analogous method to other AGN jets or to M87 at other epochs, we must derive a more general condition.

On these grounds, Hirotani et al. (1999) generalized the condition $N_e^* B^2 > 2\delta_{\rm max}^{-2}$ and applied it to the 3C 279 jet on parsec scales. In that paper they revealed that the core and components C3 and C4, of which spectra are obtained from the literature, are dominated by a pair plasma. It is interesting to note that the same conclusion that the 3C 279 jet is dominated by a pair plasma is derived from an independent method by Wardle et al. (1998), who studied the circularly polarized radio emission from the 3C 279 jet.

In the present paper we apply the same method to the 3C 345 jet. The quasar 3C 345 (redshift $z = 0.594$) is one of a class of core-dominated flat-spectrum radio sources that are believed to emit X-rays via the synchrotron self-Compton process. VLBI imaging observations of the "superluminal" quasar 3C 345 have been made at 5 GHz every year since 1977 (Unwin & Wehrle 1992), while 10.5 and 22 GHz observations have occurred at more frequent intervals (e.g., Biretta, Moore, & Cohen 1986). The apparent speeds of components C2, C3, C4, and C5 increase monotonically with time from $\sim 3c$ to $\sim 10c$, consistent with a jet of constant Lorentz factor ($\Gamma = 10$) bending away from the line of sight (Zensus, Cohen, & Unwin 1995). Later, Unwin et al. (1997) studied the time evolution of spectral shapes and angular sizes of component C7 at a distance ~ 0.5 mas (2 pc) from the nucleus. Using the physical parameters given in the literature above and deducing the kinetic luminosity from its core-position offset, we conclude that all five jet components are likely dominated by an $e^{\pm}$ plasma. In § 4 we discuss the validity of these assumptions.

We use a Hubble constant $H_0 = 65\, h$ km s^{-1} Mpc^{-1} and $q_0 = 0.5$ throughout this paper. These give a luminosity

[1] Present address: National Astronomical Observatory, Osawa, Mitaka 181-8588, Japan.

distance to 3C 345 of $D_L = 3.06\ h^{-1}$ Gpc. An angular size or separation of 1 mas corresponds to 5.83 h^{-1} pc. A proper motion of 1 mas yr^{-1} translates into a speed of $\beta_{\rm app} = 30.3\ h^{-1}$. Spectral index α is defined such that $S_\nu \propto \nu^\alpha$.

2. CONSTRAINTS ON MAGNETIC FLUX AND PARTICLE NUMBER DENSITIES

We shall distinguish whether a radio-emitting component is dominated by a normal plasma or an $e^\pm$ plasma by imposing two independent constraints on N_e^*. First, in § 2.1 we give the synchrotron self-absorption constraint, which is obtained by extending the work by Reynolds et al. (1996) (see Appendix A). Second, in § 2.2 the kinematic luminosity constraint is presented.

2.1. *Synchrotron Self-Absorption Constraint*

In this paper we model a jet component with redshift z as homogeneous spheres of angular diameter θ_d, containing a tangled magnetic field B (G) and relativistic electrons that give a synchrotron spectrum with optically thin index α and maximum flux density S_m (Jy) at frequency ν_m. We can then compute the magnetic field density as follows (Cohen 1985; Ghisellini et al. 1992):

$$B = 10^{-5} b(\alpha) S_m^{-2} (m)^5 \left(\frac{\theta_d}{\rm mas}\right)^4 \frac{\delta}{1+z} , \qquad (1)$$

where δ is the beaming factor defined by

$$\delta \equiv \frac{1}{\Gamma(1-\beta \cos \varphi)} , \qquad (2)$$

$\Gamma \equiv 1/(1-\beta^2)^{1/2}$ is the bulk Lorentz factor of the jet component moving with velocity βc, and φ is the orientation of the jet axis to the line of sight. The coefficient $b(\alpha)$ is given in Cohen (1985). Both Γ and φ can be uniquely computed from δ and $\beta_{\rm app}$ as follows:

$$\Gamma = \frac{\beta_{\rm app}^2 + \delta^2 + 1}{2\delta} , \qquad (3)$$

$$\varphi = \tan^{-1}\left(\frac{2\beta_{\rm app}}{\beta_{\rm app}^2 + \delta^2 - 1}\right) . \qquad (4)$$

We assume that the electron number density between energies E and $E + dE$ is expressed by a power law as

$$\frac{dN_e^*}{dE} = N_0 E^{2\alpha - 1} . \qquad (5)$$

Integrating dN_e^*/dE from $\gamma_{\rm min} m_e c^2$ to $\gamma_{\rm max} m_e c^2$ and assuming $\gamma_{\rm max} \gg \gamma_{\rm min}$ and $\alpha < 0$, we obtain the electron number density

$$N_e^* = \frac{\gamma_{{\rm min}2\alpha}}{-2\alpha} (m_e c^2)^{2\alpha} N_0 . \qquad (6)$$

Computing the optical depth along the line of sight, Marscher (1983) expressed N_0 in terms of θ_d, S_m, ν_m, and α. Combining the result with equation (6), we finally obtain (see also Appendix B)

$$N_e^{*\,({\rm SSA})} = e(\alpha) \frac{\gamma_{\rm min}^{2\alpha}}{-2\alpha} \frac{h(1+z)^2 q_0^2 \sin \varphi}{zq_0 + (q_0 - 1)(-1 + \sqrt{2q_0 z + 1})} \times \left(\frac{\theta_d}{\rm mas}\right)^{4\alpha - 7} \left(\frac{\nu_m}{\rm GHz}\right)^{4\alpha - 5} S_m^{-2\alpha + 3} \left(\frac{\delta}{1+z}\right)^{2\alpha - 3} , \qquad (7)$$

where $e(\alpha) \equiv 2.39 \times 10^{1-6.77\alpha}$ $(0 < -\alpha < 1.25)$. If the component is not resolved enough, this equation gives the lower bound of N_e^*.

2.2. *Kinetic Luminosity Constraint*

As described in Appendix B in detail, we can infer the kinetic luminosity, $L_{\rm kin}$, from the core-position offset, $\Omega_{r\nu}$, due to synchrotron self-absorption. For the core, we assume a conical geometry with a small half-opening angle χ. Then $L_{\rm kin}$ measured in the rest frame of the AGN becomes

$$L_{\rm kin} \sim C_{\rm kin} K \frac{r_1^2}{r_0^3} \beta\Gamma(\Gamma - 1)\chi^2 \left(\frac{\Omega_{r\nu}/\nu_0}{r_1 \sin \varphi}\right)^{2(5-2\alpha)/(7-2\alpha)} \times \left[\pi C(\alpha) \frac{\chi}{\sin \varphi} \frac{K}{\gamma_{\rm min}} \frac{r_1}{r} \frac{-2\alpha}{\gamma_{\rm min}^{2\alpha}} \left(\frac{\delta}{1+z}\right)^{3/2-\alpha}\right]^{-4/(7-2\alpha)} , \qquad (8)$$

where K is defined by equation (B13) and becomes 0.1 for $\alpha = -0.5$ if an energy equipartition holds between the radiating particles and the magnetic field.

For a pure pair plasma, we obtain $C_{\rm kin} = \pi^2 \langle \gamma_- \rangle m_e c^3/\gamma_{\rm min}$, where $\langle \gamma_- \rangle$ is the averaged Lorentz factor of randomly moving electrons and positrons, which could be computed from equation (5) for a power-law distribution of radiating particles. For a normal plasma, on the other hand, we obtain $C_{\rm kin} = \pi^2 m_p c^3/(2\gamma_{\rm min})$, where m_p refers to the rest mass of a proton. It should be noted that $\gamma_{\rm min}$ takes a different value from a pair plasma.

Once $L_{\rm kin}$ of a stationary jet is obtained, we can deduce N_e^* at an arbitrary position along the jet, even if the geometry deviates from a cone. When the jet has a perpendicular half-width $R_\perp$ at a certain position, $L_{\rm kin}$ and N_e^* are related by

$$L_{\rm kin} = \pi R_\perp^2 \beta c \Gamma N_e^* (\Gamma - 1)(\langle \gamma_- \rangle m_e c^2 + \langle \gamma_+ \rangle m_+ c^2) , \qquad (9)$$

where $\langle \gamma_- \rangle$ and $\langle \gamma_+ \rangle$ refer to the averaged Lorentz factors of electrons and positively charged particles, respectively; m_+ designates the mass of the positive charge. Replacing the angular diameter distance, $2R_\perp/\theta_d$, with the luminosity distance divided by $(1+z)^2$, we can solve equation (9) for N_e^* to obtain

$$N_e^{*\,({\rm kin})} = \frac{3.42 \times 10^2 h^2 q_0^4 (1+z)^4}{[zq_0 + (q_0 - 1)(-1 + \sqrt{2q_0 z + 1})]^2} \times \left(\frac{\theta_d}{\rm mas}\right)^{-2} \frac{1}{\beta\Gamma(\Gamma - 1)} \frac{L_{46.5}}{\langle \gamma_- \rangle + \langle \gamma_+ \rangle m_+/m_e} \ {\rm cm}^{-3} , \qquad (10)$$

where $L_{46.5}$ refers to the kinetic luminosity in the unit of $10^{46.5}$ ergs s^{-1}. It should be noted that $\langle \gamma_- \rangle + \langle \gamma_+ \rangle m_+/m_e$ becomes roughly $2\gamma_{\rm min} \ln(\gamma_{\rm min}/\gamma_{\rm max})$ for a pair plasma with $\alpha \sim -0.5$, while it becomes 1836 for a normal plasma. As a result, $N_e^{*\,({\rm kin})}$ for a pair plasma becomes about $100\gamma_{\rm min}^{-1}$ times greater than that for a normal plasma. Since $N_e^{*\,({\rm SSA})}$ is proportional to $\gamma_{\rm min}^{2\alpha}$, the ratio $N_e^{*\,({\rm kin})}/N_e^{*\,({\rm SSA})}$ for a pair plasma becomes about $100\gamma_{\rm min}^{-1-2\alpha}$ times greater than that for a normal plasma. For a jet component close to the VLBI core, we may put $\alpha \sim -0.5$; therefore, the dependence on $\gamma_{\rm min}$ virtually vanishes.

In short, we can exclude the possibility of a normal plasma dominance if $1 < N_e^{*\,({\rm pair})}/N_e^{*\,({\rm SSA})} \ll 100$ is satisfied, where $N_e^{*\,({\rm pair})}$ refers to the value of $N_e^{*\,({\rm kin})}$ computed for a pair plasma. On the other hand, $N_e^{*\,(\mathrm{pair})}/N_e^{*\,(\mathrm{SSA})} < 1$ implies that L_{kin} is underestimated. The conclusion is invulnerable against the value of γ_{min} of electrons and positrons.

3. APPLICATION TO THE 3C 345 JET

Let us apply the method described above to the 3C 345 jet on parsec scales and investigate the matter content. It is, however, difficult to define α, ν_m, and S_m of each component well because the spectral information for an individual component is limited by the frequency coverage and quality of VLBI measurements near a given epoch. Therefore, Zensus et al. (1995) chose self-consistent values that matched the data and gave a reasonable fit to the overall spectrum when the components C2, C3, and C4 (hereafter C2–C4) and the core are considered together (Table 1). For C2 and C3, they used the highest value for ν_m, while for C4 they used a representative possibility. Subsequently, Unwin et al. (1997) obtained these radio parameters for C5 and C7 by analogous method. We present these parameters together with their errors in Table 2. The jet half-opening angle $\chi \sim 2\overset{\circ}{.}4$ is calculated from measuring the jet size within 1 mas distance from the core (§ 4.3 in Lobanov 1998). We choose $\alpha = -0.65$ as the spectral index of the core below the turnover frequency at 700 GHz (§ 5.2 of Zensus et al. 1995).

TABLE 1

MAGNETIC FIELD AND ELECTRON DENSITY OF COMPONENTS C2–C4

Parameter	C2 (1982.0)	C3 (1982.0)	C4 (1982.0)
ρh (mas)[a]	4.9/0.65	2.2/0.65	0.40/0.65
$\beta_{\mathrm{app}} h$[a]	8.4/0.65	6.0/0.65	4.0/0.65
ν_m (GHz)[a]	1.5	2.6	14.6
S_m (Jy)[a]	2.0	2.1	7.6
α[a]	−0.6	−0.7	−0.3
θ_d (mas)[a]	2.15	0.97	0.29
δ_{min}[a]	2.1	3.6	14.3
δ_{eq}[b]	6.7	13	17
Γ_{eq}[b]	16	9.8	9.6
φ_{eq} (rad)[b]	0.12	0.072	0.039
B (mG)[b]	5.7	6.9	18
$N_e^{*\,(\mathrm{SSA})}$ (cm^{-3})[b]	0.11	0.33	0.19
$N_e^{*\,(\mathrm{pair})}$ (cm^{-3})[b]	$0.095L_{46.5}$	$1.3L_{46.5}$	$16L_{46.5}$
$N_e^{*\,(\mathrm{pair})}/N_e^{*\,(\mathrm{SSA})}$[b]	$0.86L_{46.5}$	$4.0L_{46.5}$	$80L_{46.5}$
$e^{\pm}$ dominated?	Likely yes	Likely yes	Maybe yes

[a] From Zensus et al. 1995.

[b] The values for $h = 1.0$ are presented. Kinetic luminosity is normalized as $L_{46.5} \equiv L_{\mathrm{kin}}/10^{46.5}$ ergs s^{-1} (see text).

3.1. *Kinetic Luminosity*

To estimate the kinetic luminosity from equation (8), we have to input Γ, φ, $\Omega_{r\nu}$, and δ for a given C_{kin}, K, χ, and α. Let us first consider Γ, φ, and δ. As demonstrated in Figure 4 in Unwin et al. (1997), a component (C7) accelerated as it moved away from the core, the Lorentz factor increased from $\Gamma \sim 5$ to $\Gamma > 10$, and the viewing angle increased from $\varphi \sim 2°$ to $\varphi \sim 10°$. It is inappropriate to consider the case $\varphi \ll \chi$; therefore, we assume $\varphi \sim 2°$ for the core. In this case $\delta \gg 1$ holds to give $L_{\mathrm{kin}} \propto \Gamma(\Gamma - 1)/\delta \propto \delta$. In the case of a newly born component (C7) at 1992.05 Unwin et al. (1997) derived a conservative limit $\delta > 11.7$ by assuming that C7 was the origin of the observed X-rays. Therefore, it is likely that δ is much greater than 10 for the core because δ decreased as the component moved away.

The core-position offset of the 3C 345 jet was reported by Lobanov (1998), who derived the reference value $\Omega_{r\nu} = 10.7$ pc Hz^{-1}. For a pair plasma with $\alpha \sim -0.5$, $\langle \gamma_- \rangle \sim \gamma_{\mathrm{min}} \ln(\gamma_{\mathrm{max}}/\gamma_{\mathrm{min}})$ holds in the expression of C_{kin}; therefore, equation (8) gives

$$L_{\mathrm{kin}} \sim 10^{46}\, \frac{\ln(\gamma_{\mathrm{max}}/\gamma_{\mathrm{min}})}{10}\, K^{0.5} \left(\frac{\delta}{20}\right) \text{ ergs s}^{-1} . \tag{11}$$

On the other hand, for a normal plasma, equation (8) gives

$$L_{\mathrm{kin}} \sim 10^{46} \left(\frac{\gamma_{\mathrm{min}}}{100}\right)^{-1} K^{0.5} \left(\frac{\delta}{20}\right) \text{ ergs s}^{-1} . \tag{12}$$

Unless the particles significantly dominate the magnetic field, $K^{0.5}$ does not exceed unity (see eqs. [B14] and [B15], which hold when an energy equipartition is realized between the radiating particles and the magnetic field). For

TABLE 2

MAGNETIC FIELD AND ELECTRON DENSITY OF COMPONENTS C5 AND C7

Parameter	C5 (1990.55)	C7 (1992.05)	C7 (1992.67)	C7 (1993.19)	C7 (1993.55)
ρh (mas)[b]	1.75/0.65	0.14/0.65	0.22/0.65	0.38/0.65	0.52/0.65
$\beta_{\mathrm{app}} h$	5.7/0.65 [a]	1.8/0.65 [b]	3.9/0.65 [b]	6.8/0.65 [b]	9.4/0.65 [b]
ν_m (GHz)[b]	2.7 ± 0.5	12.8 ± 0.5	12.5 ± 1.0	11.6 ± 0.5	11.0 ± 1.5
S_m (Jy)[b]	3.2 ± 0.5	4.6 ± 0.5	7.0 ± 0.5	5.1 ± 0.5	3.1 ± 0.5
α[b]	−0.75	−0.75	−0.75	−0.75	−0.75
θ_d (mas)[b]	0.80	$0.20 \pm .04$	$0.35 \pm .02$	$0.41 \pm .02$	$0.38 \pm .02$
δ_{min}[b]	8.0 ± 3.5	11.7 ± 4.1	6.5 ± 0.9	5.5 ± 0.6	4.0 ± 1.1
δ_{eq}[c]	33	39^{+43}_{-24}	15^{+6}_{-5}	$8.4^{+2.5}_{-2.3}$	$7.1^{+6.7}_{-4.7}$
Γ_{eq}[c]	18	20^{+21}_{-12}	$8.8^{+2.6}_{-2.1}$	$11.0^{+0.8}_{-0.8}$	25^{+19}_{-10}
φ_{eq} (rad)[c]	0.022	$0.008^{+0.014}_{-0.007}$	$0.05^{+0.04}_{-0.03}$	$0.12^{+0.02}_{-0.03}$	$0.11^{+0.02}_{-0.04}$
B (mG)[c]	4.4	19^{+11}_{-11}	31^{+12}_{-12}	43^{+12}_{-12}	62^{+61}_{-39}
$N_e^{*\,(\mathrm{SSA})}$ (cm^{-3})[c]	0.09	$0.5^{+1.9}_{-0.4}$	$4.5^{+6.6}_{-2.9}$	11^{+5}_{-4}	12^{+10}_{-8}
$N_e^{*\,(\mathrm{pair})}$ (cm^{-3})[c]	$0.63L_{46.5}$	$11^{+17}_{-8}L_{46.5}$	$14^{+7}_{-6}L_{46.5}$	$5.9^{+1.4}_{-1.8}L_{46.5}$	$1.9^{+1.9}_{-1.6}L_{46.5}$
$N_e^{*\,(\mathrm{pair})}/N_e^{*\,(\mathrm{SSA})}$[c]	$7.2L_{46.5}$	$23^{+40}_{-12}L_{46.5}$	$3.0^{+1.9}_{-1.2}L_{46.5}$	$0.53^{+0.44}_{-0.27}L_{46.5}$	$0.17^{+0.81}_{-0.15}L_{46.5}$
$e^{\pm}$ dominated?	Likely yes	Likely yes	Likely yes	Likely yes	Likely yes

[a] From Unwin & Wehrle 1992.

[b] From Unwin et al. 1997. Errors are nominally 1 σ but are dominated by systematic errors, which are included in the estimate.

[c] The values for $h = 1$ are presented. $L_{46.5} \equiv L_{\mathrm{kin}}/10^{46.5}$ ergs s^{-1}. Errors are 90% confidence regions for a single parameter of interest.

a normal plasma jet, the energy distribution must cut off at $\gamma_{\rm min} \sim 100$ (§ 4; see also Wardle et al. 1998). Since $\delta > 100$ is unlikely for the 3C 345 jet, we adopt $L_{\rm kin} = 10^{46.5}$ ergs s^{-1} (or equivalently $L_{46.5} = 1$) as the representative upper bound in this paper. If $L_{\rm kin}$ becomes less than this value, the possibility of normal plasma dominance further decreases.

3.2. *Equipartition Doppler Factor*

We estimate the value of δ by assuming an energy equipartition between the magnetic field and the radiating particles. In this case K becomes of the order of unity, and δ is given by the so-called equipartition Doppler factor (Readhead 1994):

$$\delta = \delta_{\rm eq} \equiv \left\{\left[\frac{10^3 F(\alpha)}{(\theta_d/{\rm mas})}\right]^{34}\left[\frac{2(h/1.54)}{1-1/\sqrt{1+z}}\right]^2 \times (1+z)^{15-2\alpha} S_{\rm m}^{16}\left(\frac{\nu_m}{\rm MHz}\right)^{-35-2\alpha}\right\}^{1/(13-2\alpha)}, \quad (13)$$

where $F(\alpha)$ is given in Scott & Readhead (1977).

There is much justice in adopting the equipartition Doppler factor as the representative value. First, as Güijosa & Daly (1996) pointed out, $\delta_{\rm eq}$ values of various AGN jets have a high correlation with $\delta_{\rm min}$, the minimum allowed Doppler factor derived by comparing the predicted and the observed self-Compton flux (Marscher 1983, 1987; Ghisellini et al. 1992). (If a homogeneous moving sphere emits all the observed X-ray flux via synchrotron self-Compton process, then $\delta = \delta_{\rm min}$.) Second, the ratio $\delta_{\rm eq}/\delta$ weakly depends on the ratio u_p/u_B, where u_p and u_B refer to the energy densities of radiating particles (i.e., electrons and positrons) and the magnetic field, respectively. For $\alpha = -0.75$ for instance, we obtain $\delta_{\rm eq}/\delta = (u_p/u_B)^{2/17}$ (Readhead 1994). It is noteworthy that $N_e^{*\,({\rm SSA})}$ depends relatively weakly on θ_d, ν_m, and α if we adopt $\delta = \delta_{\rm eq}$. For example, we obtain $N_e^{*\,({\rm SSA})} \propto \theta_d^{2.9}\nu_m^{5.5}S_m^{-1.5}$ for $\alpha = -0.75$. This forms a striking contrast with $N_e^{*\,({\rm SSA})} \propto \theta_d^{-10}\nu_m^{-8}S_m^{4.5}\delta^{-4.5}$, which would be obtained from equation (7) without making any assumptions on δ. We present such representative values of $\delta_{\rm eq}$, $\Gamma_{\rm eq} \equiv (\beta_{\rm app}^2 + \delta_{\rm eq}^2 + 1)/(2\delta_{\rm eq})$, B, and $N_e^{*\,({\rm SSA})}$ for C2–C4 in Table 1, and those for C5 and C7 in Table 2.

We first compare the values of $\delta_{\rm eq}$ with $\delta_{\rm min}$. It follows from Tables 1 and 2 that $\delta_{\rm eq} > \delta_{\rm min}$ is satisfied for all the eight cases, as expected. Moreover, the values of $\delta_{\rm eq}$ for C2–C4 at 1982.0 and those for C7 at the four epochs decrease with increasing projected distance, ρ (mas), from the core. As a result, the viewing angle computed from $\beta_{\rm app}$ and $\delta_{\rm eq}$ (see eq. [4]), $\varphi_{\rm eq}$, increases with increasing ρ. (We exclude C5, for which the trajectory appears in a different position angle from those for C2–C4.) The results are qualitatively consistent with Zensus et al. (1995) and Unwin et al. (1997).

Let us next consider $N_e^{*\,({\rm SSA})}$. This variable is roughly constant at ~ 0.2 cm^{-3} for C2–C4, whereas it increases from 0.5 cm^{-3} at 1992.05 to 10 cm^{-3} at 1993.55 for C7. We consider that this tendency comes from insufficient angular resolution, particularly when a component is close to the core. We can alternatively compute N_e^* from $N_e^* = (K/\gamma_{\rm min} m_e c^2)(B^2/8\pi)$, the energy equipartition. Remembering that $K \sim 0.1$ for $\alpha \sim -0.5$, we find that N_e^* computed in this way is consistent with $N_e^{*\,({\rm SSA})}$.

We can compute $N_e^{*\,({\rm pair})}$, $N_e^{*\,({\rm kin})}$ for a pair plasma from equation (8). The results of $N_e^{*\,({\rm pair})}$ are presented in Tables 1 and 2, together with the ratio $N_e^{*\,({\rm pair})}/N_e^{*\,({\rm SSA})}$. It follows from Table 1 that C2 and C3 are likely dominated by a pair plasma. It is also suggested that C4 is dominated by pair plasma unless $L_{\rm kin}$ exceeds $10^{46.5}$ ergs s^{-1}. Unfortunately, the errors in B, $N_e^{*\,({\rm SSA})}$, and $N_e^{*\,({\rm pair})}$ cannot be calculated because those in ν_m and S_m are not presented in Zensus et al. (1995). Furthermore, Table 2 indicates that C5 and C7 at all the four epochs are likely dominated by a pair plasma. Unfortunately, the meaningful errors in B, $N_e^{*\,({\rm SSA})}$, and $N_e^{*\,({\rm pair})}$ for C5 cannot be calculated because its error in θ_d (or ξ in their notation) is not presented in Unwin et al. (1997). Nevertheless, the results of $N_e^{*\,({\rm pair})}/N_e^{*\,({\rm SSA})}$ strongly suggest that the jet components of 3C 345 on parsec scales are dominated by a pair plasma.

4. DISCUSSION

In summary, we derive the proper electron number density, $N_e^{*\,({\rm SSA})}$, of a homogeneous radio-emitting component of which spectral turnover is due to synchrotron self-absorption. Comparing $N_e^{*\,({\rm SSA})}$ with the density derived from the kinetic luminosity of the jet, we can investigate whether we can exclude the possibility of normal plasma (e^--p) dominance. Applying this method to the "superluminal" quasar 3C 345, using the published spectrum data of C2, C3, C4, C5, and C7, we find that all the five components are likely dominated by a pair plasma.

As demonstrated in the last part of § 2, the conclusion is invulnerable against the undetermined value of $\gamma_{\rm min}$ of electrons and positrons. However, if $\gamma_{\rm min}$ for a *normal* plasma were to be significantly less than 100, then the possibility of a normal plasma dominance could not be ruled out in general. In the case of the 3C 345 jet, equation (12) would give $L_{\rm kin} \sim 10^{48}$ ergs s^{-1} for a normal plasma with $\gamma_{\rm min} \sim 1$. In this case, the large kinetic luminosity ($\sim 10^{48}$ ergs s^{-1}) is carried by protons, because

$$\langle\gamma_-\rangle m_e c^2 \sim \frac{\gamma_{\rm min}}{K} m_e c^2 \ll m_p c^2 \quad (14)$$

holds. Nevertheless, we consider that such a jet is unlikely because the protons carry about 2 orders of magnitude more energy than is seen to be dissipated as synchrotron radiation ($\sim 10^{46}$ ergs s^{-1}). Electrons on parsec scales will not be cooled down so rapidly shortly after being heated up at the shock fronts.

It is interesting to consider the case when δ is estimated by methods other than the energy equipartition. As an example let us consider a jet motion with a roughly constant Lorentz factor; Zensus et al. (1995) derived that $\Gamma \sim 10$ is close to the smallest value that is consistent with all their available kinematic constraints. Such values of δ and φ are denoted by the solid dots in Figure 12 of their paper and tabulated again in Table 3 in the present paper. Using those data, we can compute B and $N_e^{*\,({\rm SSA})}$ of each component (Table 3). For C2, we adopt $\Gamma = 13$ rather than 10 because $\beta_{\rm app} = 12.9$ for $h = 1$ (or equivalently $H_0 = 65$) gives $\Gamma > (1 + \beta_{\rm app}^2)^{1/2} = 12.9$. The results of $N_e^{*\,({\rm pair})}/N_e^{*\,({\rm SSA})}$ show again that C2–C4 at 1982.0 are likely dominated by a pair plasma.

TABLE 3

ELECTRON DENSITIES WHEN Γ IS GIVEN

Parameter	C2 (1982.0)	C3 (1982.0)	C4 (1982.0)
ρ (mas)	4.9/0.65	2.2/0.65	0.40/0.65
$\beta_{\rm app}h$	8.4/0.65	6.0/0.65	4.0/0.65
Γ	13	10	10
δ	14	13	18
B (mG)[a]	11	7.2	19
$N_e^{*\,\rm (SSA)}$ (cm^{-3})[a]	0.0055	0.77	1.2
$N_e^{*\,\rm (pair)}$ (cm^{-3})[a]	$0.15L_{46.5}$	$1.3L_{46.5}$	$14L_{46.5}$
$N_e^{*\,\rm (pair)}/N_e^{*\,\rm (SSA)}$[a]	$27L_{46.5}$	$1.6L_{46.5}$	$11L_{46.5}$
$e^{\pm}$ dominated?	Likely yes	Likely yes	Likely yes

[a] $h = 1$ is assumed. $L_{46.5} \equiv L_{\rm kin}/10^{46.5}$ ergs s^{-1}.

APPENDIX A

DERIVATION OF THE SYNCHROTRON SELF-ABSORPTION CONSTRAINTS

We assume that the parsec-scale jet close to the core propagates conically with a half-opening angle χ in the observer's frame. Then the optical depth τ for synchrotron self-absorption is given by

$$\tau_\nu(R) = \frac{2R \sin \chi}{\sin (\varphi + \chi)} \alpha_\nu , \tag{A1}$$

where R is the distance of the position from the injection point of the jet and α_ν (1 cm^{-1}) refers to the absorption coefficient. For a small half-opening angle ($\chi \ll 1$), this equation can be approximated as

$$\tau_\nu(R) = 2R \frac{\chi}{\sin \varphi} \alpha_\nu . \tag{A2}$$

Since τ and $R\chi$ are Lorentz invariants, we obtain

$$\frac{\alpha_\nu}{\sin \varphi} = \frac{\alpha_\nu^*}{\sin \varphi^*} , \tag{A3}$$

where a quantity with an asterisk is measured in the comoving frame while that without an asterisk in the observer's frame. Since $\nu\alpha_\nu$ is also Lorentz invariant, equation (A3) gives

$$\frac{\sin \varphi^*}{\sin \varphi} = \frac{\nu}{\nu^*} = \frac{\delta}{1+z} . \tag{A4}$$

Combining equations (A2) and (A4), we obtain

$$\tau_\nu = \frac{1+z}{\delta} \frac{2R\chi}{\sin \varphi} \alpha_\nu^* = \frac{1+z}{\delta} \frac{1}{\sin \varphi} \frac{\theta_d D_L}{(1+z)^2} \alpha_\nu^* , \tag{A5}$$

where the angular diameter distance of the jet, $2R\chi/\theta_d$, is rewritten with the luminosity distance, D_L, divided by $(1+z)^2$; here θ_d is the angular diameter of the component in the perpendicular direction of the jet propagation. If we observe τ_ν at the turnover frequency, ν_m, it becomes a function of the optically thin spectral index α, which is tabulated in Scott & Readhead (1977).

Averaging over pitch angles of the isotropic electron power-law distribution (eq. [5]), we can write down the absorption coefficient in the comoving frame as (Le Roux 1961; Ginzburg & Syrovatskii 1965)

$$\alpha_\nu^* = C(\alpha) r_0^2 k_e^* \frac{\nu_0}{\nu^*} \left(\frac{\nu_B}{\nu^*}\right)^{(-2\alpha+3)/2} , \tag{A6}$$

where $\nu_0 \equiv c/r_0 \equiv c/[e^2/(m_e c^2)]$ and $\nu_B \equiv eB/(2\pi m_e c)$. The coefficient $C(\alpha)$ is given in Table 1 of Gould (1979).

Substituting equation (A6) into equation (A5), and assuming $\alpha < 0$ and $\gamma_{\rm min} \ll \gamma_{\rm max}$, we obtain with the aid of equation (A5)

$$N_e^* B^{-\alpha+1.5} = \frac{m_e c}{e^2} \left(\frac{e}{2\pi m_e c}\right)^{-1.5+\alpha} \frac{\tau_\nu(\alpha)}{C(\alpha)} \frac{\gamma_{\rm min}^{2\alpha}}{-2\alpha} \times \frac{(1+z)^2}{D_L} \frac{\sin \varphi}{\theta_d} \left(\frac{1+z}{\delta}\right)^{-\alpha+1.5} \nu^{-\alpha+2.5} . \tag{A7}$$

Evaluating ν at the turnover frequency, $\nu = \nu_m$, and combining with equation (1), we obtain N_e^* presented in equation (7), which equals $(\gamma_{\rm min} m_e c^2)^{2\alpha}/(-2\alpha)$ times N_0 given in equation (3) in Marscher (1983). It is noteworthy that electron number density in the observer's frame can be obtained if we multiply $(1+z)/\delta$ on N_e^*.

APPENDIX B

KINETIC LUMINOSITY INFERRED FROM CORE-POSITION OFFSET

In this appendix we deduce the kinetic luminosity of a jet from its core-position offset due to synchrotron self-absorption. This method was originally developed by Lobanov (1998). However, our results somewhat differ from his results; therefore, we explicitly describe the derivation so that the readers can check it.

B1. *Scaling Law*

First, we assume that N_e^* and B scale on r in the following manner:

$$N_e^* = N_1 r^{-n}, \quad B = B_1 r^{-m} , \tag{B1}$$

where N_1 and B_1 refer to the values of N_e^* and B at $r_1 = 1$ pc, respectively; $r \equiv R/r_1$. Introducing dimensionless variables

$$x_N \equiv r_1 r_0^2 N_1, \quad x_B \equiv \frac{\nu_{B_1}}{\nu_0} = \frac{eB_1}{2\pi m_e c} \tag{B2}$$

and utilizing equation (A6), we obtain from the left equality in equation (A5)

$$\tau_\nu = C(\alpha) \frac{2\chi}{\sin\varphi} \frac{-2\alpha}{\gamma_{\min}^{2\alpha}} \left(\frac{1+z}{\delta}\right)^{-\epsilon} \left(\frac{\nu}{\nu_0}\right)^{-1-\epsilon} r^{1-n-m\epsilon} x_N x_B^{\epsilon} , \tag{B3}$$

where $\epsilon \equiv 3/2 - \alpha$.

At a given frequency ν, the flux density will peak at the position where τ_ν becomes unity. Thus setting $\tau = 1$ and solving equation (B3) for r, we obtain the distance from the VLBI core observed at frequency ν from the central engine as

$$r(\nu) = \left(x_B^{k_b} F \frac{\nu_0}{\nu}\right)^{1/k_r} , \tag{B4}$$

where

$$F(\alpha) \equiv \left[C(\alpha) \frac{2\chi}{\sin\varphi} \frac{-2\alpha}{\gamma_{\min}^{2\alpha}} \left(\frac{\delta}{1+z}\right)^{\epsilon} x_N\right]^{1/(\epsilon+1)} , \tag{B5}$$

$$k_b \equiv \frac{3-2\alpha}{5-2\alpha} , \tag{B6}$$

$$k_r \equiv \frac{(3-2\alpha)m + 2n - 2}{5-2\alpha} . \tag{B7}$$

B2. *Core-Position Offset*

If we measure $r(\nu)$ at two different frequencies (say ν_a and ν_b), equation (B4) gives the dimensionless, projected distance of $r(\nu_a) - r(\nu_b)$ as

$$\Delta r_{\text{proj}} = [r(\nu_a) - r(\nu_b)] \sin\varphi = (x_B^{k_b} F\nu_0)^{1/k_r} \frac{\nu_b^{1/k_r} - \nu_a^{1/k_r}}{\nu_a^{1/k_r}\nu_b^{1/k_r}} \sin\varphi . \tag{B8}$$

Defining the core-position offset as

$$\Omega_{r\nu} \equiv r_1 \Delta r_{\text{proj}} \frac{\nu_a^{1/k_r}\nu_b^{1/k_r}}{\nu_b^{1/k_r} - \nu_a^{1/k_r}} , \tag{B9}$$

we obtain

$$\frac{\Omega_{r\nu}}{r_1} = (x_B^{k_b} F\nu_0)^{1/k_r} \sin\varphi . \tag{B10}$$

To express x_B in terms of x_N and $\Omega_{r\nu}$, we can invert equation (B10) as

$$x_B = \left(\frac{\Omega_{r\nu}}{r_1 \sin\varphi}\right)^{k_r/k_b} (F\nu_0)^{-1/k_b} . \tag{B11}$$

Note that x_N is included in $F = F(\alpha)$.

Setting $\nu_b \to \infty$ in equation (B8), we obtain the absolute distance of the VLBI core measured at ν from the central engine as

$$r_{\text{core}}(\nu) = \frac{\Omega_{r\nu}}{r_1 \sin\varphi} \nu^{-1/k_r} . \tag{B12}$$

That is, once Ω_{rv} is obtained from multifrequency VLBI observations, we can deduce the distance of the synchrotron self-absorbing VLBI core from the central engine, assuming the scaling laws of N_e^* and B as equation (B1).

We next represent x_N and x_B (or equivalently, N_1 and B_1) as a function of Ω_{rv}. To this end, we relate N_e^* and B as follows:

$$N_e^* \gamma_{\min} m_e c^2 = K \frac{B^2}{8\pi} . \tag{B13}$$

When an energy equipartition between the radiating particles and the magnetic field holds, equation (5) gives for $\alpha = -0.5$:

$$K = \frac{1}{\ln(\gamma_{\max}/\gamma_{\min})} \sim 0.1 , \tag{B14}$$

whereas for $\alpha < -0.5$

$$K = \frac{2\alpha + 1}{2\alpha} \frac{\gamma_{\max}^{2\alpha} - \gamma_{\min}^{2\alpha}}{\gamma_{\max}^{2\alpha+1} - \gamma_{\min}^{2\alpha+1}} . \tag{B15}$$

Substituting $N_e^* = N_1 r^{-2}$ and $B = B_1 r^{-1}$ into (B13) and replacing N_1 and B_1 with x_N and x_B, we obtain

$$x_N = \frac{\pi}{2} \frac{K}{\gamma_{\min}} \frac{r_1}{r_0} x_B^2 . \tag{B16}$$

It is noteworthy that the assumptions of $n = 2$ and $m = 1$, which results in $k_r = 1$, guarantees the energy equipartition at an arbitrary distance, r.

Combining equations (B11) and (B16), we obtain

$$x_B = \left(\frac{\Omega_{rv}/\nu_0}{r_1 \sin\varphi}\right)^{(5-2\alpha)/(7-2\alpha)} \times \left[\pi C(\alpha) \frac{\chi}{\sin\varphi} \frac{K}{\gamma_{\min}} \frac{r_1}{r_0} \frac{-2\alpha}{\gamma_{\min}^{2\alpha}} \left(\frac{\delta}{1+z}\right)^{\epsilon}\right]^{-2/(7-2\alpha)} . \tag{B17}$$

The particle number density, x_N, can be readily computed from equation (B16).

B3. *Kinetic Luminosity*

We can now relate the kinetic luminosity with the core-position offset. The factor $N_e R^2$ in equation (9) can be expressed in terms of x_N and hence x_B as

$$N_e^* R^2 = N_1 r_1^2 = \frac{r_1}{r_0^2} x_N = \frac{\pi}{2} \frac{K}{\gamma_{\min}} \frac{r_1^2}{r_0^3} x_B^2 . \tag{B18}$$

For a pure pair plasma, we obtain $\langle\gamma_+\rangle = \langle\gamma_-\rangle$ and $m_+ = m_e$. Therefore, for a conical geometry, we can put $R_\perp = R\chi$ in equation (9) to obtain equation (8), where $C_{\mathrm{kin}} = \pi^2 \langle\gamma_-\rangle m_e c^3/\gamma_{\min}$.

In the same manner, for a normal plasma, we have $\langle\gamma_+\rangle = 1$ and $m_+ = m_p$. In this case we obtain $C_{\mathrm{kin}} = \pi^2 m_p c^3/(2\gamma_{\min})$.

REFERENCES

Biretta, J. A., Moore, R. L., & Cohen, M. H. 1986, ApJ, 308, 93
Cohen, M. H. 1985, in Proc. Winter School on Extragalactic Energy Sources, ed. V. K. Kapahi (Bangalore: Indian Acad. Sci.), 1
Despringre, V., & Fraix-Burnet, D. 1997, A&A, 320, 26.
Ghisellini, G., Padovani, P., Celotti, A., & Maraschi, L. 1992, MNRAS, 258, 776
Ginzburg, U. L., & Syrovatskii, S. I. 1965, ARA&A, 3, 297
Gómez, J. L., Alberdi, A., & Marcaide, J. M. 1993, A&A, 274, 55
———. 1994a, A&A, 284, 51
———. 1994b, A&A, 292, 33
Gould, R. J. 1979, A&A, 76, 306
Güijosa, A., & Daly, R. A. 1996, ApJ, 461, 600
Hirotani, K., Iguchi, S., Kimura, M., & Wajima, K. 1999, PASJ, 51, 263
Le Roux, E. 1961, Ann. d'Astrophys., 24, 71
Lobanov, A. P. 1998, A&A, 330, 79
Marscher, A. P. 1983, ApJ, 264, 296
Marscher, A. P. 1987, in Superluminal Radio Sources, ed. J. A. Zensus, T. J. Pearson, & M. H. Cohen (Cambridge: Cambridge Univ. Press), 280
Pauliny-Toth, I. I. K., Preuss, E., Witzel, A., Graham, D., Kellerman, K. I., & Ronnang, B. 1981, AJ, 86, 371
Pelletier, G., & Sol, H. 1992, MNRAS, 254, 635
Readhead, A. C. S. 1994, ApJ, 426, 51
Reynolds, C. S., Fabian, A. C., Celotti, A., & Rees, M. J. 1996, MNRAS, 283, 873
Scott, M. A., & Readhead, A. C. S. 1977, MNRAS, 180, 539
Soll, H. Pelletier, G., & Asséo, E. 1989, MNRAS, 237, 411
Unwin, S. C., & Wehrle, A. E. 1992, ApJ, 398, 74
Unwin, S. C., Wehrle, A. E., Lobanov, A. P., Zensus, J. A., & Madejski, G. M. 1997, ApJ, 480, 596
Wardle, J. F. C., Homan, D. C., Ojha, R., & Roberts, D. H. 1998, Nature, 395, 457
Zensus, J. A., Cohen, M. H., & Unwin, S. C. 1995, ApJ, 443, 35